

α -particle condensate states in ^{16}O

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Abstract

The existence of a rotational band with the $\alpha+^{12}\text{C}(0_2^+)$ cluster structure, in which three α particles in $^{12}\text{C}(0_2^+)$ are locally condensed, is demonstrated near the four- α threshold of ^{16}O in agreement with experiment. This is achieved by studying structure and scattering for the $\alpha+^{12}\text{C}(0_2^+)$ system in a unified way. A drastic reduction (quenching) of the moment of the inertia of the 0^+ state at 15.1 MeV just above the four- α threshold in ^{16}O suggests that it could be a candidate for the superfluid state in α -particle condensation.

Keywords: α -particle condensation, $\alpha+^{12}\text{C}$ scattering, α -cluster structure, ^{16}O , superfluid

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α -particle condensation has been paid much attention in light nuclei. Up to now the 0_2^+ (7.65 MeV) Hoyle state of ^{12}C has been considered to be a candidate for a three α -particle condensate with a dilute density distribution [1]. The elaborate microscopic α -cluster model wave functions of the Hoyle state by Uegaki et al. [2] and Kamimura et al. [3], which reproduce many experimental data involving the Hoyle state are almost completely equivalent to the condensate wave function [4]. Many theoretical studies [1, 5, 6, 7, 8, 9] support the dilute property of the Hoyle state. However, the typical physical modes such as superfluidity and/or a quantum vortex have not been observed.

The fundamental question that may arise is that superfluidity due to α -particle condensation is difficult to observe in nuclear systems like ^{12}C and ^{16}O , while superfluidity has clearly been observed in bulk systems such as He II and ^3He liquids. We note, however, that recent studies of parahydrogen [10] and He clusters [11] show that superfluidity can be observed in small systems with 10 or less particles. This encourages us to study the superfluidity of a small number of α particles in strong-interaction systems composed of protons and neutrons. One of the most convincing ways to demonstrate the existence of α -particle condensation is to show

superfluidity of the system. It has been shown [12] theoretically and experimentally that a reduction (quenching) of the moment of inertia from the rigid-body value is characteristic to the superfluid behavior of a dilute Bose gas due to the occurrence of Bose-Einstein condensation. This reduction is also observed for liquid helium (the Hess-Fairbank effect) [13]. Path Integral Monte Carlo (PIMC) is a simulation which makes use of this reduction of the moment of inertia from the classical moment of inertia in calculating the superfluid density, for example, of quantum fluids in confined geometries [14].

In studies of α -particle condensation in ^{16}O , so far mostly the 0^+ state has been discussed [1, 15, 16]. Tohsaki et al. [1] thought that the 0^+ state at $E_x=14.0$ MeV located below the four- α threshold energy is an α -particle condensate. Wakasa et al. and Funaki et al. [15] suggested that a newly observed 0^+ state at $E_x=13.6$ MeV is an α -particle condensate. On the other hand, very recently Funaki et al. [16] performed a semi-microscopic four- α cluster model calculation in the OCM (Orthogonality Condition Model) and concluded that the 0^+ state at $E_x=15.1$ MeV can probably be an α -particle condensate. These states were shown to have dilute density distributions in the frame of the *bound state approximation*. A dilute density distribution is not equivalent to α -particle condensation and no clear experimental evidence for α -particle condensation such as superfluidity and/or vortex excitation has been

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presented. α -particle condensation in ^{16}O has been controversial. We note that in experiment in the high excitation energy region above the four- α threshold well-developed α -cluster states (2^+ , 4^+ and 6^+) have been observed in the $^8\text{Be}+^8\text{Be}$ and $\alpha+^{12}\text{C}(0_2^+)$ decay channels of ^{16}O [17, 18, 19, 20]. These states have been considered as linear chain states of four α particles for many years [17]. We think that it is important to understand not only the resonant 0^+ state but also the other resonant higher spin states built on it in a unified way in the context of α -particle condensation. It is also important to study the α decaying resonant states above the α threshold by solving the *scattering equation* correctly.

In this paper, from a unified description of structure and scattering for the $\alpha+^{12}\text{C}$ system, we show that a rotational band with the $\alpha+^{12}\text{C}(0_2^+)$ cluster structure is predicted near the four- α threshold in ^{16}O . The above α -cluster states observed in the four- α decay channel [17, 18, 19, 20] can be understood consistently as fragmented states of the band. It is shown that the observed 0^+ state at $E_x=15.1$ MeV just above the four- α threshold, which we interpret to be fragmented from the broad band head 0^+ state, has a reduced moment of inertia compared to the well-developed $\alpha+^{12}\text{C}(0_2^+)$ cluster structure. It is suggested that this 0^+ state could be a candidate for the superfluid state in ^{16}O in four α -particle condensation.

In α -cluster studies a unified description of structure and scattering has been powerful because the interaction potential can be uniquely determined from rainbow scattering [21]. In fact, a unified study of structure and scattering of the $\alpha+^{40}\text{Ca}$ system could disentangle a long-standing controversy about the α -cluster structure in ^{44}Ti [22, 23]. This unification may be extended to the case where a target nucleus is excited because the nuclear rainbow and prerainbow appear also in inelastic scattering and the mechanism can be understood in a similar way to elastic scattering [24, 21].

We study the elastic and inelastic $\alpha+^{12}\text{C}$ scattering, and states with the $\alpha+^{12}\text{C}$ cluster structure in a unified way using a double folding (DF) model in the coupled channel method by taking into account the excited states of ^{12}C , which has been shown to be successful in describing α and ^3He scattering at the high and low energy regions [8, 9]. The diagonal and coupling potentials of the DF model for the α - ^{12}C system are calculated as follows:

$$V_{ij}(\mathbf{R}) = \int \rho_{00}^{(\alpha)}(\mathbf{r}_1) \rho_{ij}^{(\text{C})}(\mathbf{r}_2) v_{\text{NN}}(E, \rho, \mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2, \quad (1)$$

where $\rho_{00}^{(\alpha)}(\mathbf{r})$ is the ground state density of the α particle, while v_{NN} denotes the density dependent M3Y ef-

fective interaction (DDM3Y) [25] usually used in the DF model. $\rho_{ij}^{(\text{C})}(\mathbf{r})$ represents the diagonal ($i = j$) or transition ($i \neq j$) nucleon density of ^{12}C which is calculated using the resonating group method by Kamimura et al. [3]. In the calculation of densities of ^{12}C , the shell-like structure of the ground state 0_1^+ and the well-developed α -cluster structure of the Hoyle 0_2^+ state are simultaneously well reproduced. These wave functions have been checked against many experimental data including charge form factors, electric transition probabilities and reactions involving excitation to the 0_2^+ state [3]. The wave function for the Hoyle 0_2^+ state is very close to the α -condensate wave function. In the calculations the normalization factor $N_R + iN_I$ is introduced for the α - ^{12}C DF potential. The imaginary part takes into account absorption phenomenologically.

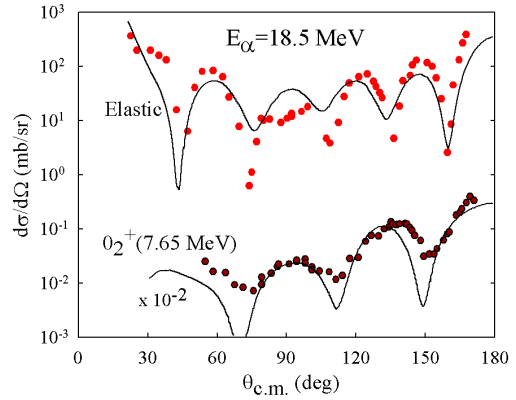


Figure 1: Calculated angular distributions (solid lines) in elastic and inelastic $\alpha+^{12}\text{C}$ scattering at $E_\alpha=18.5$ MeV are compared with the experimental data (points) [26].

We have shown in Ref.[8] that elastic and inelastic $\alpha+^{12}\text{C}$ rainbow scattering from the Hoyle state in the high energy region ($E_\alpha=139, 166$ and 172.5 MeV) can be well described in the DF model with $N_R=1.23$ - 1.26 . We extend this analysis to the lowest energy $E_\alpha=18.5$ MeV where experimental data of both inelastic scattering from the Hoyle state and elastic scattering are available [26]. We analyze the angular distributions in the coupled channel calculations including most of the channels of ^{12}C open at this energy, that is, g.s. $2_1^+(4.44$ MeV), $3^-(9.65$ MeV), $0_2^+(7.65$ MeV) and $2_2^+(10.3$ MeV). The absorption due to the coupling to all the other open channels, i.e., $p+^{15}\text{N}$, $n+^{15}\text{O}$ and $d+^{14}\text{N}$ channels, is introduced as a small imaginary potential with $N_I=0.045$. The calculated angular distributions with $N_R=1.398$ are compared with the experimental data in Fig. 1. The volume integral per nucleon pair for the real potential is $J_V=427.3$ MeVfm 3 for the g.s. channel.

The characteristic oscillations at the backward hemisphere of the experimental data are well reproduced by the calculation. The backward rise in elastic scattering, ALAS (Anomalous Large Angle Scattering), is caused by the internal waves [27], which penetrate deep into the internal region of the potential. The ALAS seen for inelastic scattering from the Hoyle state is also understood similarly in terms of internal waves [24]. These results suggest that the diagonal and coupling potentials in Eq. (1) work in the α -cluster structure study in the low energy region near the $\alpha+^{12}\text{C}(0_2^+)$ threshold.

We study the resonant α -cluster structure of ^{16}O by solving the coupled channel scattering equations with use of the real part of the double folding potential. We take $N_R=1.34$, which is chosen so that the calculated energy of the band head 1^- state of the $K=0_1^-$, which has a well-developed $\alpha+^{12}\text{C}(\text{g.s.})$ structure, corresponds well with experimental energy. The volume integral J_V and rms radius of the potential are 409.6 MeVfm³ and 3.44 fm for the g.s. channel, and 510.3 MeVfm³ and 4.29 fm for the $\alpha-^{12}\text{C}(0_2^+)$ channel, respectively. The energy-dependence of N_R , that is, $N_R=1.23$ at $E_\alpha=139$ MeV [8], $N_R=1.389$ at $E_\alpha=18.5$ MeV and $N_R=1.34$ at $E_\alpha \approx 0$, which increases toward the lower energy as the incident energy decreases and again increases toward zero energy, seems to be consistent with the threshold anomaly.

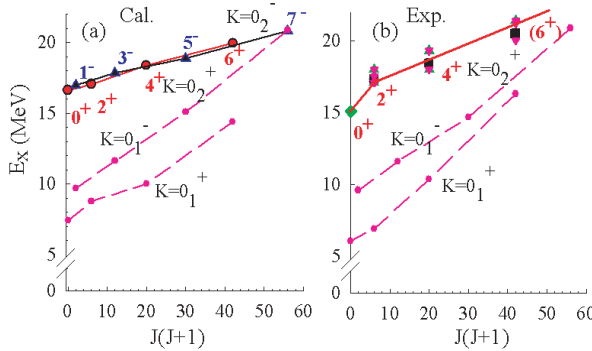


Figure 2: (a) Calculated states of the $K=0_1^+$ and $K=0_1^-$ parity-doublet bands with the $\alpha+^{12}\text{C}(\text{g.s.})$ structure, and the $K=0_2^+$ and $K=0_2^-$ bands with the $\alpha+^{12}\text{C}(0_2^+)$ cluster structure near the four- α threshold are compared with (b) the experimental levels in ^{16}O taken from Ref.[17] (triangle up), Ref.[18, 19, 20] (triangle down) and Ref.[28] (diamond). The centroid of each of the spin states is shown by a square. The lines are to guide the eye.

In Fig 2 the calculated states are shown in comparison with the experimental data. The calculation reproduces the parity-doublet $K=0_1^+$ band and $K=0_1^-$ band with the $\alpha+^{12}\text{C}(\text{g.s.})$ structure well. A resonance energy E_r is defined as the energy where the elastic channel phase

shift δ_J passes $\pi/2$. (The 0^+ state of the $K=0_1^+$ band is well below the barrier and calculated in the single channel bound state approximation.) In Table I the calculated excitation energy of resonant states of the parity-doublet $K=0_1^+$ and $K=0_1^-$ bands and its widths derived from $\Gamma_\alpha = 2/\frac{d\delta_J}{dE} \Big|_{E=E_r}$ are displayed in comparison with the experimental data [28]. Because the calculated widths are strongly dependent on excitation energy, the dimensionless reduced widths θ_α^2 , which are more physically related to the degree of α -clustering, are also shown at the three channel radii $a=5.2, 5.6$ and 6.0 fm. θ_α^2 is defined as $\Gamma_\alpha = 2P(a)\gamma^2(a)$ with $P(a)$ being the penetration factor, $\gamma^2(a) = \theta_\alpha^2(a)\gamma_w^2(a)$ and the Wigner limit $\gamma_w^2(a) = 3\hbar^2/2\mu a$ with μ denoting the reduced mass. The agreement of the calculated θ_α^2 with experiment is good especially for the $K=0_1^-$ band states. The large θ_α^2 reconfirms that the $K=0_1^-$ band has a well-developed α -cluster structure. The rather small calculated reduced widths for the positive parity states may be improved by introducing a small parity-dependence in the potential in order to reproduce the experimental excitation energy [29].

In Fig. 2 the α -cluster structure, in which ^{12}C is excited to the Hoyle state, is shown. These resonant states with positive parity form a $K=0_2^+$ band structure. The resonant structure with the $\alpha+^{12}\text{C}(0_2^+)$ is seen by investigating the S-matrix and partial wave cross sections from the ground state to the Hoyle state. The modulus of the S-matrix $S_{g.s.-0_2^+}$, which indicates the transition strength of the incident flux to the Hoyle state, shows a peak at the energy which corresponds to a resonant state of the composite $\alpha+^{12}\text{C}(0_2^+)$ system. Accordingly the partial wave cross section also shows a peak at the energy corresponding to the peak of the S-matrix. In Fig. 3(a) the partial cross sections calculated in the coupled five channels are displayed. In Fig. 3(b) the partial cross sections calculated in the two channels (g.s. and 0_2^+) are shown to help to understand the origin of the peaks. In the two channel calculations prominent peaks are seen clearly, which corresponds to a resonance of the composite system with the $\alpha+^{12}\text{C}(0_2^+)$ cluster structure. In the five channel calculations each peak is fragmented. The resonant structure with the $\alpha+^{12}\text{C}(0_2^+)$ cluster $K=0_2^-$ band is also obtained for negative parity states. In Table II the excitation energy of the calculated resonant states of the parity-doublet $K=0_2^+$ and $K=0_2^-$ bands and its widths are displayed. These are evaluated from the partial cross sections from the ground state to the 0_2^+ state assuming a Breit-Wigner function. As seen in Fig. 2, quite remarkably our potential, which reproduces the higher energy rainbow scattering and low

Table 1: The calculated excitation energy, α -decay width Γ_α and dimensionless reduced width $\theta_\alpha^2(a)$ of the resonant states of the $K = 0_1^+$ and $K = 0_1^-$ bands are compared with the experimental data [28]. $\theta_\alpha^2(a)$ is evaluated at the channel radii $a=5.2, 5.6$ and 6.0 fm.

	J^π	cal.						exp.			
		E_x (MeV)	Γ_α (keV)	$\theta_\alpha^2(a)$ a (fm)			E_x (MeV)	Γ_α (keV)	$\theta_\alpha^2(a)$ a (fm)		
				5.2	5.6	6.0			5.2	5.6	6.0
$K = 0_1^+$	4^+	10.00	5	0.15	0.09	0.06	10.36	26 ± 3	0.35	0.22	0.14
	6^+	14.37	34	0.10	0.06	0.04	16.28	420 ± 20	0.36	0.26	0.20
$K = 0_1^-$	1^-	9.67	778	1.02	0.87	0.79	9.59	480 ± 20	0.71	0.61	0.54
	3^-	11.61	822	0.64	0.54	0.48	11.60	800 ± 100	0.63	0.53	0.47
	5^-	15.09	440	0.26	0.21	0.18	14.66	672 ± 11	0.49	0.38	0.31
	7^-	20.86	1790	0.77	0.59	0.49	20.86	904 ± 55	0.39	0.30	0.25

energy scattering and the lowest α -cluster states of the parity-doublet $K = 0_1^+$ and $K = 0_1^-$ bands, locates the $K = 0_1^+$ band with the $\alpha + {}^{12}\text{C}(0_2^+)$ cluster structure near the $\alpha + {}^{12}\text{C}(0_2^+)$ threshold ($E_x = 14.82$ MeV), namely near the four- α threshold ($E_x = 14.44$ MeV). It is noted that our predicted $K = 0_2^+$ band states as well as the $K = 0_2^-$ band states are on the rotational $J(J+1)$ trajectory. The Pauli principle is not important in this highly excited energy region. As for the $K = 0_2^-$ band, the observed states, 1^- at $E_x = 16.20$ MeV with $\Gamma = 0.58$ MeV and 5^- at $E_x = 18.40$ MeV with $\Gamma = 0.55$ MeV could be a candidate. No 3^- state has been reported at the corresponding excitation energy. More experimental studies are needed for the negative parity states including the coincidence experiments. We have investigated the N_R dependence of the location of the $K = 0_2^+$ band. In calculations the band head energies are $E_x = 16.9, 16.6,$ and 16.9 MeV for $N_R = 1.30, 1.34,$ and 1.398 , respectively, which shows that the N_R -dependence is mild in the relevant region.

Table 2: The calculated excitation energies and widths of the $K = 0_2^+$ and $K = 0_2^-$ band states.

J^π	$K = 0_2^+$			J^π	$K = 0_2^-$	
	E_x (MeV)	Γ (MeV)			E_x (MeV)	Γ (MeV)
0^+	16.61	1.14		1^-	16.98	0.57
2^+	17.04	0.45		3^-	17.83	0.27
4^+	18.38	0.23		5^-	18.86	0.24
6^+	19.95	0.39		7^-	20.81	1.17

The $2^+, 4^+$ and 6^+ states of the $K = 0_2^+$ band shown in Fig. 2(b) were observed in the coincidence experiments to search for the four α -particle states [17, 18, 19, 20]. In more detail, Chevallier et al. [17] first observed the 2^+ (16.95 MeV), 2^+ (17.15 MeV), 4^+ (18.05 MeV) and 6^+ (19.35 MeV) states in the ${}^{12}\text{C}(\alpha, {}^8\text{Be})$ reaction, which is interpreted to lie on the rotational band with a

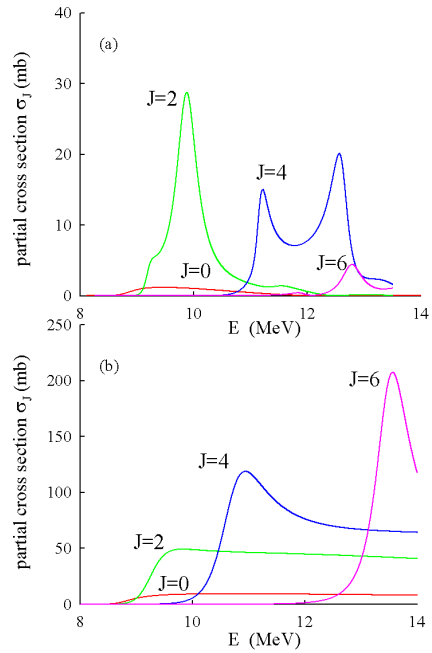


Figure 3: Partial wave cross sections in inelastic scattering from the Hoyle state in the coupled channel calculations are displayed as a function of the incident energy of the α particle in the center of mass system E , (a) coupled five channels (g.s., 2_1^+ , 3^- , 0_2^+ and 2_2^+) and (b) coupled two channels (g.s. and 0_2^+). The vertical scales of (a) and (b) are different.

very small rotational constant $k = \hbar^2/2I = 64$ keV where I is the moment of inertia. They concluded from this very large moment of inertia that “the only conceivable structure with such a moment of inertia is of four α ’s laid out in a string and rotating rigidly” and thereafter the band has been considered to be a linear chain for more than four decades. Later, Freer et al. performed the ${}^{12}\text{C}({}^{16}\text{O}, 4\alpha)$ reaction [18] and observed the two 2^+ (17.1 and 17.5 MeV), 2^+ or 4^+ (18.0 MeV), 4^+ (19.3 MeV) and (6^+) (21.4 MeV) states, which they

interpreted to lie on the rotational band with a small $k=95\pm 20$ keV. Furthermore Freer et al. observed the high spin states, two 6^+ states at 20.0 and 21.2 MeV in the $^{12}\text{C}(^{12}\text{C}, ^8\text{Be}+^8\text{Be})$ reaction [19, 20]. We interpret that the α strength of all the above states observed in the coincidence experiments may be considered to be fragmented from a well-developed cluster band and the centroid of each of the spin states is displayed in Fig. 2(b). (The fragmentation of the α strength is caused by the coupling between the genuine cluster band state and other states nearby, which has been demonstrated experimentally and theoretically in the typical α -cluster structure in ^{40}Ca and ^{44}Ti [23, 30].) The rotational constant estimated from each centroid of the 2^+ , 4^+ and 6^+ states is $k=86$ keV, which is also very small consistent with the above values by Freer et al. [18] and Chevallier et al. [17]. The very large moment of inertia, about four times that of the $K = 0_1^+$ band built on the 0^+ state at 6.06 MeV, which has an $\alpha+^{12}\text{C}(\text{g.s.})$ cluster structure, implies a very extended structure of ^{16}O in these states. Our calculated $k=80$ keV is close to the experimental value $k=86$ keV. The large moment of inertia is caused by the mechanism that the α particles are locally condensed in the Hoyle state with an extended density distribution. The moment of inertia [31] classically calculated from the matter distribution used in Eq. (1) shows that 56% of the large total moment of inertia, which is given by $I = I(\alpha) + I(^{12}\text{C}(0_2^+)) + I(\text{rel.})$ where $I(\alpha)$, $I(^{12}\text{C}(0_2^+))$ and $I(\text{rel.})$ stand for the moment of inertia of the α particle, $^{12}\text{C}(0_2^+)$ and their relative motion, respectively, comes from $I(^{12}\text{C}(0_2^+))$, i.e., the extended α -particle distribution in the Hoyle state. 40% of I comes from $I(\text{rel.})$, from which the intercluster distance R between the orbiting α particle and the $^{12}\text{C}(0_2^+)$ core is estimated to be $R=5.9$ fm. Considering that the superfluid part of $^{12}\text{C}(0_2^+)$ due to α -particle condensation, whose probability is about 70% [5, 6], does not contribute to $I(^{12}\text{C}(0_2^+))$, the $I(\text{rel.})$ becomes larger, that is, the intercluster distance increases significantly. If we write the total moment of inertia as $I=\mu R^2$, we get $R=9.2$ fm, which is very large, although smaller than 12.3 fm in the α -particle linear chain picture given in Ref.[17], suggesting an α -particle halo configuration. It is very interesting and important to measure the size of the states of the $K = 0_2^+$ band experimentally. The fact that the member states, the two 2^+ (17.1 and 17.5 MeV), (2^+ or 4^+) (18.0 MeV) and 4^+ (19.3 MeV) states, are observed in the $\alpha+^{12}\text{C}(0_2^+)$ decay channel [18] are also reasonably understood by our picture that the band has an $\alpha+^{12}\text{C}(0_2^+)$ cluster structure instead of the linear chain structure of the four α particles. Since the channel coupling between the Hoyle state and the 2_2^+ state, which

has a well-developed $\alpha+^8\text{Be}$ cluster structure as well as the Hoyle state [2], is strong in the present calculation, the $^8\text{Be}+^8\text{Be}$ configuration might also contribute to the deformation. The fact that the band states are observed in the $^{12}\text{C}(\alpha, ^8\text{Be})$ reaction seems to be consistent with this possibility.

What is especially interesting is the 0^+ state. Our calculation locates the band head 0^+ state at around 16.6 MeV. However, in the above-mentioned coincidence experiments no 0^+ state was observed at the predicted energy region. In the compilation of the energy levels of ^{16}O in Refs.[28, 32] no 0^+ state with a large α -decay width has been observed near 16.6 MeV. (A state reported at 16.36 MeV (0^+ , 1^-) in Ref.[33], which could be the same state as the 0^+ state at 16.33 MeV observed in the (p,t) reaction [32], has a very small α -decay width of $\Gamma_\alpha/\Gamma=0.07$.) The reported states with a considerable α width in Refs.[28, 32] observed in α -particle scattering from ^{12}C are 15.1 MeV (0^+), 17.6 MeV (0^+ or 1^-), and 18.1 MeV (0^+ , spin tentative) with $\Gamma_\alpha/\Gamma=0.35$, 0.32, and 0.31, respectively. Because the calculated band head 0^+ state is very broad with no centrifugal barrier as seen in Fig. 3(b), it may have escaped from detection. The above observed states nearby may be considered to be fragmented from the band head 0^+ state. The centroid of the above three states is 16.9 MeV, which is close to our predicted energy.

If a four- α condensate 0^+ state exists near the threshold, it will be located below the calculated band head 0^+ state with the $\alpha+^{12}\text{C}(0_2^+)$ configuration because the moment of inertia is reduced. As seen in Fig. 2(b), the moment of inertia of the 0^+ state at 15.1 MeV with a large α width below the calculated band head energy and just above the four- α threshold is drastically reduced to a quarter of that of the 2^+ and 4^+ states of the $K = 0_2^+$ band. Because the orbiting α particle is also sitting in the $0s$ state like the three α particles in the $0s$ state in $^{12}\text{C}(0_2^+)$, this could be a good candidate state in which four α particles are condensed spherically in the lowest $0s$ state. The probability of condensation is roughly conjectured to be three quarters from the reduction of the moment of inertia. This is consistent with a recent calculation by Funaki et al. [16] who claim that the probability that the four α particles are sitting in the $0s$ state is 61%. This 0^+ state is less dilute than the deformed $K = 0_2^+$ band states. The excited 2^+ , 4^+ and 6^+ states of the $K = 0_2^+$ band are created by lifting an α particle to the D , G and I states, respectively, which brings about the deformed $\alpha+^{12}\text{C}(0_2^+)$ configuration. Similar excited states are expected in ^{12}C . In fact, the observed 2_2^+ state has been discussed to be an α condensate state [6, 7]. However, a 4_2^+ state and a rotational band built

on the Hoyle state have not been observed in ^{12}C .

Next we discuss possible α -particle condensation in the $^{16}\text{O}\sim^{20}\text{Ne}$ region. A weak coupling holds in the α -cluster structure in the $^{16}\text{O}\sim^{20}\text{Ne}$ region [34]. The weak coupling works for the states with a more developed cluster structure like the $K = 0_2^+$ band in ^{16}O . Therefore a band with the $X+^{12}\text{C}(0_2^+)$ cluster structure such as $^8\text{Be}+^{12}\text{C}(0_2^+)$, analogous to the $K = 0_2^+$ band in ^{16}O , may be expected near the $^{12}\text{C}(0_2^+)$ threshold in ^{20}Ne , ^{19}F , ^{18}O , and ^{17}O . It is interesting to study whether the two fragmented 10^+ states at $E_x=35.2$ and 36.5 MeV in ^{20}Ne observed in the $^8\text{Be}+^{12}\text{C}(0_2^+)$ decay channel [20] could be such an analogous rotational band state. The local condensation of α particles like the $^{12}\text{C}(\text{g.s.})+^{12}\text{C}(0_2^+)$ and $^{12}\text{C}(0_2^+)+^{12}\text{C}(0_2^+)$ cluster structures as well as six α -particle condensation in ^{24}Mg is also interesting.

The idea of (local) α -particle condensation may be extended to heavier nuclei. Yamada et al. [35] discussed possible α -particle condensation in nuclei not heavier than ^{40}Ca by solving the Gross-Pitaevski and Hill-Wheeler equations. Gridnev et al. [36] discussed the fragmented α -cluster states in ^{32}S observed in ALAS of α particles from ^{28}Si from the viewpoint of α -particle condensation. Ogloblin [37] and Oertzen [38, 39] suggested an α -particle condensate in heavier nuclei. Local α -particle condensation will not be limited to ^{16}O and might be anticipated in other heavier nuclei including the $A=212$ region. The detection method proposed in Ref.[39] may be useful.

To summarize, in agreement with experiment we have shown that a rotational band with a developed $\alpha+^{12}\text{C}(0_2^+)$ cluster structure, in which three α particles are locally condensed, is located near the four- α threshold. This was achieved by using the double folding potential which reproduces elastic and inelastic α -particle scattering from ^{12}C . The moment of inertia of the 0^+ state at 15.1 MeV just above the four- α threshold is drastically reduced suggesting that this state could be a candidate for the superfluid state of α -particle condensation in ^{16}O . It would also be interesting to study the local condensation of di-neutrons, deuterons and even ^{16}O in heavier nuclei.

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